__Lemma 1__ Binomial coeffs satisfy $\binom{n}{k} = \binom{n}{n-k}$ $\forall\, n,k \in \mathbb{N}^0$ s.t. $0 \le k \le n$.

__Proof__ $(y+x)^n = \sum_{k=0}^{n} \binom{n}{k} y^k x^{n-k} = \sum_{k=0}^{n} \binom{n}{k} \boxed{x^{n-k} y^k}$

$$(y+x)^3 = (y+x)(y^2+2yx+x^2) = y^3 + 3y^2x + 3yx^2 + x^3$$

$$\binom{3}{0}=1 \quad \binom{3}{1}=3 \quad \binom{3}{2}=3 \quad \binom{3}{3}=1$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = \sum_{k=0}^{n} \binom{n}{n-k} \boxed{x^{n-k} y^k}$$

~~let $j = n-k$~~

equate coeffs

$$\Rightarrow \binom{n}{k} = \binom{n}{n-k} \quad \forall\, k \text{ s.t. } 0 \le k \le n. \quad \square$$

$$\binom{4}{0} y^4 + \binom{4}{1} xy^3 + \binom{4}{2} x^2 y^2 + \binom{4}{3} x^3 y + \binom{4}{4} x^4$$

__Lemma 2__ Binomial coeffs satisfy

(i) $\forall\, n \in \mathbb{N}^0$, $\binom{n}{0} = 1 = \binom{n}{n}$.

(ii) $\forall\, n,k \in \mathbb{N}$ s.t. $1 \le k \le n$, $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$