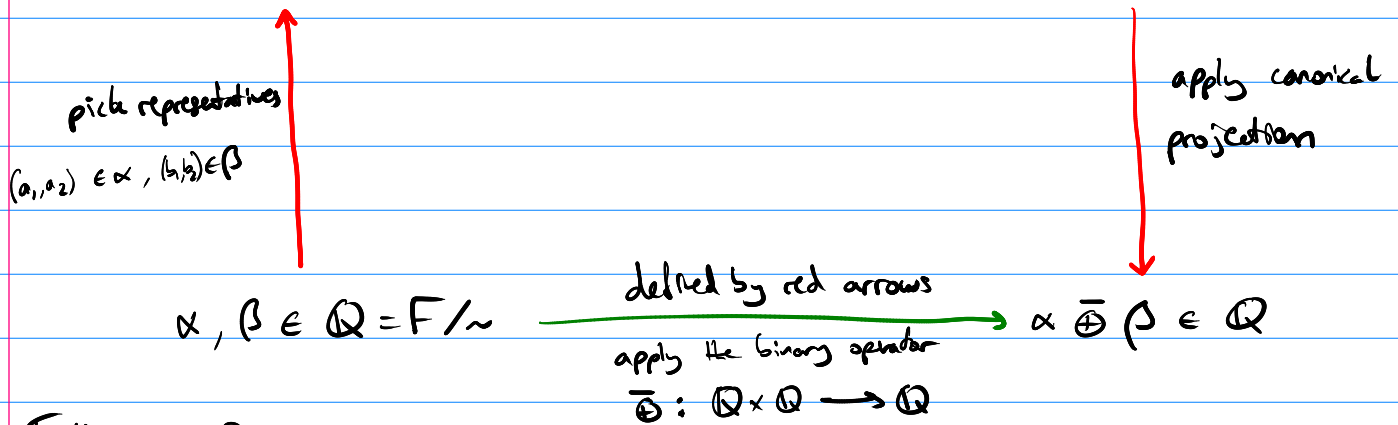


$$+ : \mathbb{Z}^2 \rightarrow \mathbb{Z}$$

$$(a, b), (x, y) \in F = \mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$$

$$(a, b) \oplus (x, y) = (ay + bx, by) \quad \mathbb{Q} = F / \sim$$

$$(a_1, a_2), (b_1, b_2) \in F \xrightarrow{\text{apply the binary operator } \oplus : F \times F \rightarrow F} (a_1 b_2 + a_2 b_1, a_2 b_2) \in F$$



Field axiom 2

Proof WTS $\alpha \bar{\oplus} \beta = \beta \bar{\oplus} \alpha$. $(a_1, a_2) \in \alpha, (b_1, b_2) \in \beta$

$$\alpha \bar{\oplus} \beta \ni (a_1 b_2 + a_2 b_1, a_2 b_2) = (b_1 a_2 + b_2 a_1, b_2 a_2) \in \beta \bar{\oplus} \alpha$$

$$\Rightarrow \alpha \bar{\oplus} \beta = \beta \bar{\oplus} \alpha. \quad \square$$

$$(a, b) \oplus (x, y) = (ax, by)$$

$$\downarrow$$

$$\alpha \bar{\oplus} \beta$$