

$$\frac{1}{2} \sim \frac{2}{4} \sim \frac{7}{14}$$

$$\text{Fractions} = \mathbb{Z} \times \mathbb{Z} \setminus \{0\} = F$$

$$\alpha = \{(1, 2), (2, 4), (7, 14), (-1, -2), \dots\}$$

$$\beta = \{(2, 3), (-2, 3), (6, 9), (-6, 9), \dots\}$$

$$(x, y) \sim (a, b) \iff xb = ya$$

$$\text{Define } Q = F / \sim$$

$$\frac{x}{y} \oplus \frac{a}{b} = \frac{xb + ya}{yb}$$

$$\alpha + \beta$$

+ addition in \mathbb{Z} ,

\oplus addition in F

$$(x, y) \oplus (a, b) = (xb + ya, yb) \in F$$

$$\alpha \oplus \beta \ni (x, y) \oplus (a, b) \quad \text{where } (x, y) \in \alpha \quad (a, b) \in \beta$$