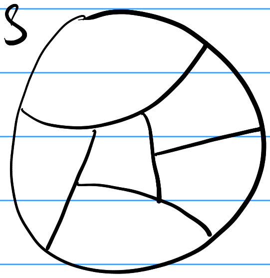
set S equivalence $\sim$ on S.

S



$S/\sim$ is the set of equivalence classes of $\sim$ on S.

If $M, N \in S/\sim$ then $M \neq \emptyset$

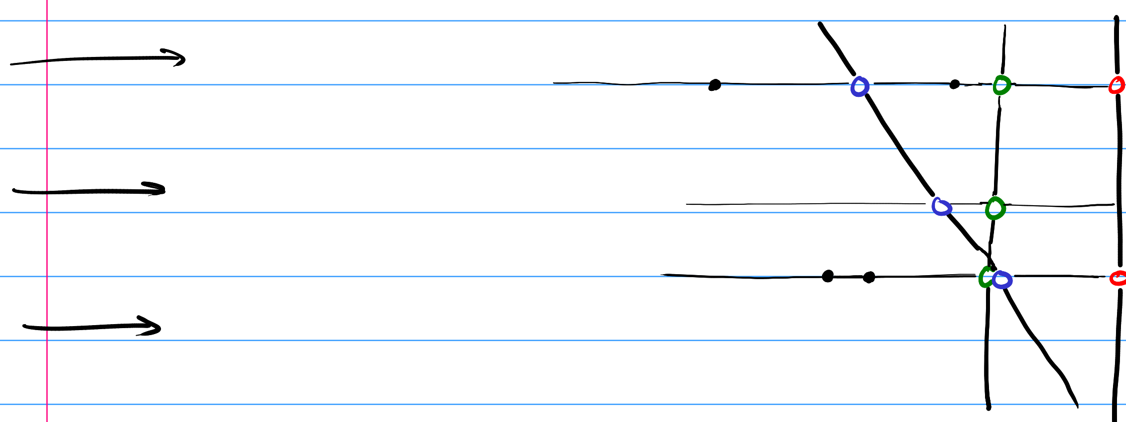
$M = N$ or $M \cap N = \emptyset$.

$\bigcup S/\sim \; = S$

$p : S \to S/\sim$ defined by $p(x) =$ the equivalence class to which x belongs

ie $p(x) \ni x$.

S



$\alpha : \mathbb{N}^0 \to \{2, 4\}$ defined by

$$\alpha(x) = 3 + (-1)^x$$

$$= \begin{cases} 3 + (-1)^2 & \text{if } x \text{ is even} \\ 3 + (-1)^1 & \text{if } x \text{ is odd} \end{cases}$$

$$= \begin{cases} 4 & \text{if } x \text{ is even} \\ 2 & \text{if } x \text{ is odd} \end{cases}$$

$\alpha(\text{even}) = 4$, $\alpha(\text{odd}) = 2$

$\exists\ \bar{\alpha} : \mathbb{N}^\circ/\sim \ \to \{2, 4\}$

$\bar{\alpha}(\text{even}) = 4$, $\bar{\alpha}(\text{odd}) = 2$.

$\bar{\alpha}$ is the <u>descent</u> of α

<u>Proof of Descent of a function theorem</u>

Define $\bar{f} : S/\sim \ \to T$ by

- Pick $x \in M$ $\qquad\qquad y \in M$
- $\bar{f}(M) = f(x)$. $\qquad \bar{f}(M) = f(y)$

Because $x \sim y \Rightarrow f(x) = f(y)$, so this is a valid defn of $\bar{f}$.

WTS $\bar{f} \circ p = f$ ie $\forall z \in S$, $(\bar{f} \circ p)(z) = f(z)$

$p : S \to S/\sim \qquad \bar{f} : S/\sim \ \to T$

$\bar{f} \circ p : S \to T$,

same domain & codomain as f ✓

LHS $= \bar{f}(p(z)) = \bar{f}(M)$ where $z \in M$
$= f(x)$ where $x \in M$
$= f(z)$.

WTS unicity. Suppose $\bar{g} : S/\sim \ \to T$ and <u>$\bar{g} \circ p = f$</u>. Aim to show $\bar{g} = \bar{f}$

ie $\forall M \in S/\sim$, $\bar{g}(M) = \bar{f}(M)$.

If $M \in S/\sim$, then $\exists\, x \in M$

$$\bar{g}(M) = \bar{g}(p(x)) = (\bar{g} \circ p)(x) = f(x) = (\bar{f} \circ p)(x) = \bar{f}(p(x))$$

$$= \bar{f}(M). \qquad \square$$